Higher-Order Co-Simulation of Field-Circuit Coupled Eddy-Current Problems

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***Abstract*—This paper discusses the weak coupling of nonlinear magnetoquasistatic field models (e.g. of induction motors) to an external electric network model (e.g. of a pulse-width-modulator). A piece-wise linear inductance (as lumped field model) is not sufficiently accurate if the magnetic field exhibits strong eddy current effects. To this end, we propose to represent the field model in the circuit equations by its Schur complement. This gives a better lumped model, which is fitted during the dynamic iteration. We demonstrate the link between iterations and the achievable order analytically and numerically.**

I. Introduction

We consider time-domain simulation. The extraction of (linear) lumped parameter model from electric machinery using FEM is a well established, [1], [2]. Those lumped models are typically used in Spice-like circuit simulators. In contrast to this, a strongly coupled simulation, [3], [4], solves the distributed field/circuit problem. This is necessary to account for nonlinear effects (e.g. saturation) in magnetic fields.

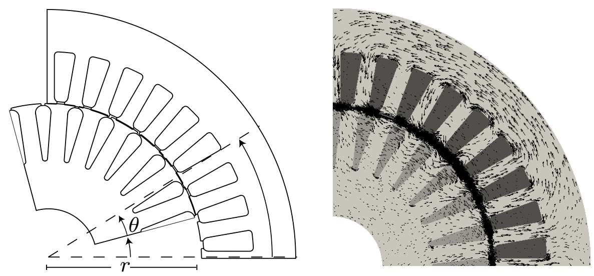
Now, the weak coupling is the synthesis of both approaches: electric circuit and magnetic field are solved independently in time, but intermediate solutions (waveforms) are exchanged at synchronization points. The machine model is then represented in the circuit by current or voltage sources, [5], or by extracted inductances, [6], [7]. This allows the circuit simulator to choose much smaller time steps than the field solver (between the synchronization points), in order to deal with switching effects in the power electronics, [8].

In this paper we abandon technically motivated circuit representations and propose more algebraic Schur complements for representing the field model. Moreover, our co-simulation explicitly allows iteration over the time windows in order to adapte the surrogate machine model to changes in the drive. We show that dynamic iteration, i.e., the repeated computation of the time windows, is necessary for any model to realize higher order time integration. For example the classical operator splitting approaches, are restricted for differential algebraic problems to first order, [9].

II. Mathematical Model

The simulation of an induction machine (Fig. 1) necessitates to consider eddy-current and saturation effects, e.g. the MQS model, [10], [11]:

$$\mathbf{M}\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\mathbf{a}} + \mathbf{k}(\widehat{\mathbf{a}},\theta) - \mathbf{X}\mathbf{i} = 0, \qquad (1)$$



(a) Rotation (b) Flux distribution

Fig. 1. 2D model of an induction motor, [2].

with circuit-coupling equations

$$\mathbf{X}^\top\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\mathbf{a}} \qquad + \mathbf{R}\mathbf{i} = \mathbf{v}, \qquad (2)$$

where $\widehat{\mathbf{a}}$ denotes the line-integrated magnetic vector potentials. $\mathbf{R}$, $\mathbf{i}$ and $\mathbf{v}$ are the resistances, currents and voltage drops of the stator windings. Each column of the matrix $\mathbf{X}$ is a discretization of a winding function, such that $\mathbf{X}\mathbf{i}$ distributes the applied currents $\mathbf{i}$. $\mathbf{M}$ is the conductivity matrix and $\mathbf{k}$ is the curl-curl-reluctivity term. The reluctivity depends nonlinearly on the magnitude of the discrete flux density $|\widehat{\widehat{\mathbf{b}}}|^2$ defined by $\widehat{\widehat{\mathbf{b}}} = \mathbf{C}\widehat{\mathbf{a}}$ (with discrete curl operator $\mathbf{C}$).

Moreover the topology depends on the rotor angle θ (see Fig. 1), which solves the motion equation:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}\theta + \lambda\frac{\mathrm{d}}{\mathrm{d}t}\theta = f(||\widehat{\widehat{\mathbf{b}}}||^2) \qquad (3)$$

with mass moment of inertia m, rotational friction λ and force f being an affine map, [10].

III. Coupling by the Schur Complement

To demonstrate this coupling, we discretize problem (1) also in time. For simplicity of notation, we use the Backward-Euler-scheme with time step Δt. In each time step for (1), the Newton iteration leads to a series of linear systems with the Jacobian

$$\mathbf{J}_a := \Big(\frac{1}{\Delta t}\mathbf{M} + \mathbf{K}(|\widehat{\widehat{\mathbf{b}}}|^2,\theta)\Big),$$

where $\mathbf{K}$ is the differential curl-curl-reluctivity matrix. Now inserting $\widehat{\mathbf{a}}$ from (1) into (2), the Jacobian (w.r.t. $\mathbf{i}$) reads

$$\mathbf{J}_i := \left(\frac{1}{\Delta t}\mathbf{L} + \mathbf{R}\right) \tag{4}$$

using the Schur complement matrix

$$\mathbf{L}(|\widehat{\mathbf{b}}|^2, \theta) := \mathbf{X}^\top \left(\frac{1}{\Delta t}\mathbf{M} + \mathbf{K}(|\widehat{\mathbf{b}}|^2, \theta)\right)^{-1} \mathbf{X}. \tag{5}$$

This results in a lumped, nonlinear, time-dependent model, which consists of a series of a generalized inductance $\mathbf{L}$ and a linear resistances $\mathbf{R}$ for each conductor. It is used as network model to represent the magnetostatic field device. Thus in every Newton-iteration of the circuit simulator, only this lumped model needs to be updated and the whole vector potential is consealed from the circuit simulator, [12]. A drawback is the additional burden of repeatedly solving the linear system for the Schur-complement. This can be reduced by bypassing, [13], or by co-simulation.

IV. Co-Simulation

Our co-simulation is organized as a Gauss-Seidel-Scheme. The time-in terval of interest $[0, T]$ is subdivided into a series of time windows: $0 = T_0 < T_1 < \cdots < T_n = T$ with synchronization points T_i. Field and circuit are sequently solved on those windows $[T_i, T_{i+1}]$, whereby each subproblem uses its own time stepping procedure (multirate, i.e, a different Δt) and whole waveforms are exchanged.

Before simulation, resistance $\mathbf{R}$ is extracted. Now, let us start by solving the field problem. To this end, we constantly extrapolate the circuit's waveforms, then solve the field and motion system problem (1–3). In a post-processing step the inductance matrix (5) is computed from the obtained waveforms. The result is inserted into the circuit equations as a lumped model. Then the circuit is solved and one obtains its waveforms. This algorithm may be applied iteratively on each time window, i.e., the field problem can be recomputed on the time window by using the new circuit waveforms etc. This is necessary, cf. [9], if a higher-order time integration is favored.

V. Higher Order Time-Integration

When using a time-integrator of order p for strong coupled problem on the time window $[T_i, T_{i+1}]$, one obtains a waveform that is accurate up to $\mathcal{O}(H_i^p)$, where $H_i = T_{i+1} - T_i$ and $H_i^p = (m \cdot \Delta t)^p$ with m time-steps. On the other hand, the weak coupling introduces additionally a splitting error. This error depends on the particular subproblems, but the order of convergence can be deduced from the abstract coupling structure, [8]. We can show by using arguments from fixed point analysis that each iteration on a time-window gives us at minimum an accuracy improvement of $\mathcal{O}(H_i)$.

VI. Conclusions

The importance of the number of iterations is also numerically visualized in Fig. 2 using the field/circuit problem from [8]. It was simulated by the weak coupling as introduced above and time-integration was carried out by RADAU5, [14]. The plot shows that each additional iteration increases the order. In this particular example by $\mathcal{O}(H^2)$. Therefore one iteration is sufficient for first order schemes like backward Euler, but more iterations are mandatory if higher order schemes are used.

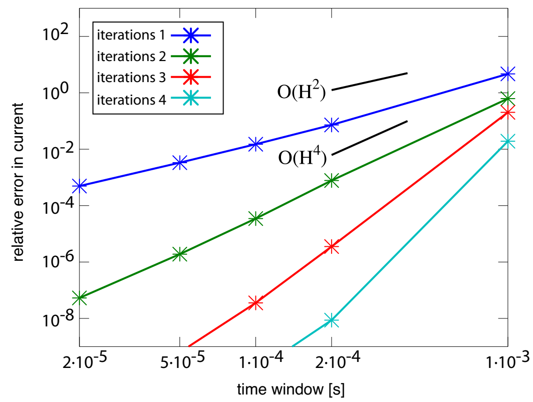


Fig. 2. The asymptotic behavior of higher order time-integaration (RADAU5) differs depending on the number of iterations.

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